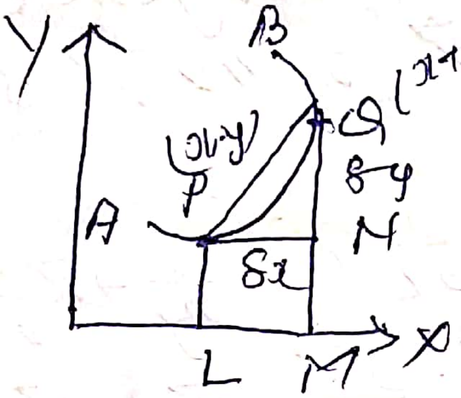
BSC (Maths) – II, paper IV

Components of velocity and Acceleration

Expression for velocity
(Cartesian coordinates)



Let us suppose that particle moving along a curve AB. At time t and $t+\delta t$ let the positions of the particle be at $P(x,y)$ and $Q(x+\delta x, y+\delta y)$. Then the chord PQ is displacement of particle in time δt.

Displacement of particle parallel to ox in time δt

$$= PN = \delta x$$

Hence the average rate of displacement parallel to x-axis $= \frac{\delta x}{\delta t}$

The rate of displacement parallel to x-axis

$$= \lim_{\delta t \to 0} \frac{\delta x}{\delta t} = \frac{dx}{dt}$$

That is, velocity at P parallel to x-axis $\frac{dx}{dt} = \dot{x}$

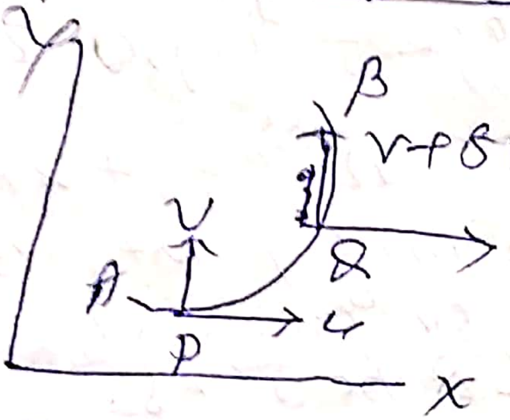
Similarly, the velocity parallel to y-axis $= \frac{dy}{dt} = \dot{y}$

If v be the magnitude of the velocity at time t (i.e. at P) and if its direction makes an angle θ with the x-axis then

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

and $\tan\theta = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{dy}{dx}$

Expression for acceleration



Let u and v be ~~compone~~ component of velocity of particle at P parallel to OX and OY respectively at time t.

~~Let~~ Let $u+\delta u$ and $v+\delta v$ be component of velocity of particle at point Q parallel to OX and OY at time $t+\delta t$

∴ The change of velocity to x-axis $= (u+\delta u) - u = \delta u$

~~Hence of~~ Average rate of change of velocity parallel to x-axis $= \frac{\delta u}{\delta t}$

Hence the rate of change of velocity parallel to the x-axis

$$= \lim_{\delta t \to 0} \frac{\delta u}{\delta t} = \frac{du}{dt}$$

i.e the resolved part of acceleration at p parallel to OX

$$= \frac{du}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \ddot{x}$$

Similarly, the resolved part of acceleration at p parallel to OY $= \frac{d^2y}{dt^2} = \ddot{y}$

If f be the magnitude of acceleration at time t (at p) and its acceleration makes an angle θ with x-axis then

$$f = \sqrt{\left(\frac{d^2x}{dt}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$$

and $\tan\theta = \frac{d^2y}{dt^2} \Big/ \frac{d^2x}{dt^2}$

$$= \frac{\ddot{y}}{\ddot{x}}$$

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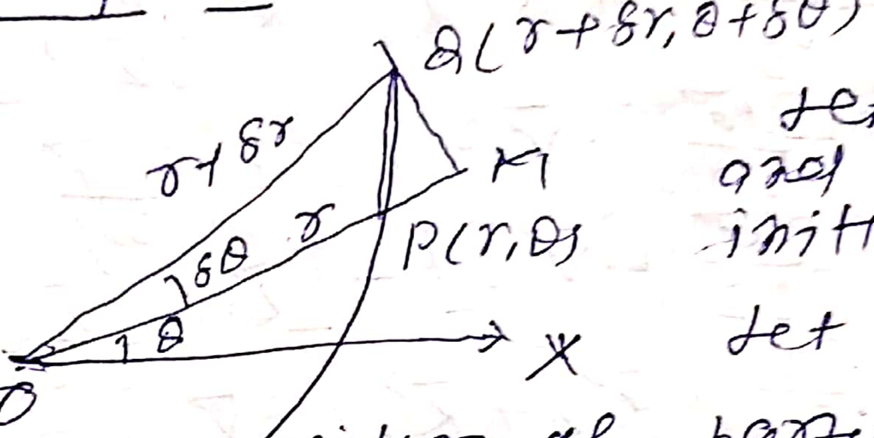
Radial and transverse velocity

Theorem prove that the radial velocity $= \frac{dr}{dt} = \dot{r}$

and transverse velocity

$= r\frac{d\theta}{dt} = r\dot{\theta}$

proof:—



Let O be pole and OX is initial line. Let $P(r,\theta)$ be the position of particle at time t and $Q(r+\delta r, \theta+\delta\theta)$ at time $t+\delta t$.

From Q draw $QM \perp OP$ (produced)

Then PM and QM are the components ~~PQ in time~~ of displacement PQ in time δt along and perpendicular to OP.

Clearly $OP = r$, $OQ = r+\delta r$,

$\angle POX = \theta$, $\angle QOX = \theta+\delta\theta$, $\angle QOP = \delta\theta$

$QM = (r+\delta r)\sin\delta\theta$, $OM = (r+\delta r)\cos\delta\theta$

Let u and v be the components of velocity of moving point along and perpendicular to OP.

Then $u = \lim_{\delta t \to 0} \dfrac{\text{displacement along OP in time } \delta t}{\delta t}$

$= \lim_{\delta t \to 0} \dfrac{PM}{\delta t} = \lim_{\delta t \to 0} \dfrac{OM - OP}{\delta t}$

$= \lim_{\delta t \to 0} \dfrac{(r+\delta r)\cos\delta\theta - r}{\delta t}$

$= \lim_{\delta t \to 0} \dfrac{(r+\delta r)\cdot 1 - r}{\delta t}$ as $\delta\theta$ is small

$= \lim_{\delta t \to 0} \dfrac{\delta r}{\delta t} = \dfrac{dr}{dt} = \dot{r}$

and $v = \lim_{\delta t \to 0} \dfrac{QM}{\delta t} = \lim_{\delta t \to 0} \dfrac{(r+\delta r)\sin\delta\theta}{\delta t}$

$= \lim_{\delta t \to 0} \dfrac{(r+\delta r)\delta\theta}{\delta r}$ as $\delta\theta$ is small

so we may write $\sin\delta\theta = \delta\theta$

$= \lim_{\delta t \to 0} r\dfrac{\delta\theta}{\delta t}$ neglecting $\delta r \delta\theta$ as it is small quantity of second order

$= r\dfrac{d\theta}{dt} = r\dot{\theta}$